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THE ECHELON SPECTROSCOPE.

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The important discovery of Zeeman of the influence of a magnetic field upon the radiations of an approximately homogeneous source shows more clearly than any other fact the great advantage of the highest attainable dispersion and resolving power in the spectroscopes employed in such observations.

If we consider that in the great majority of cases the separation of the component lines produced by the magnetic field is of the order of a twentieth to a fiftieth of the distance between the sodium lines, it will be readily admitted that, if the structure of the components themselves is more or less complex, such structure would not be revealed by the most powerful spectroscopes of the ordinary type.

In the case of the grating spectroscope, besides the difficulty of obtaining sufficient resolving power, the intensity is so feeble that only the brighter spectral lines can be observed, and even these must be augmented by using powerful discharges, — which usually have the effect of masking the structure to be investigated.

Some years ago I published a paper describing a method of analysis of approximately homogeneous radiations which depends upon the observation of the clearness of interference fringes produced by these radiations. A curve was drawn showing the change in clearness with increase in the difference of path of the two interfering pencils of light; and it was shown that there is a fixed relation between such a "visibility curve" and the distribution of light in the corresponding spectrum, — at least in the case of symmetrical lines.*

It is precisely in the examinations of such minute variations as are observed in the Zeeman effect that the advantages of this method appear, for the observations are entirely free from instrumental errors; there is practically no limit to the resolving power, and there is plenty of light.

* In the case of asymmetrical lines another relation is necessary, and such is furnished by what may be called the "phase curve."

There is, however, the rather serious inconvenience that the examination of a single line requires a considerable time, often several minutes, and during this time the character of the radiations themselves may be changing.

Besides this, nothing can be determined regarding the nature of these radiations until the "visibility curve" is complete, and analyzed either by calculation or by an equivalent mechanical operation.

Notwithstanding these difficulties, it was possible to obtain a number of rather interesting results, such as the doubling or the tripling of the central line of Zeeman's triplet, and the resolution of the lateral lines into multiple lines; also the resolution of the majority of the spectral lines examined into more or less complex groups; the observation of the effects of temperature and pressure on the width of the lines, etc.

It is none the less evident that the inconveniences of this process are so serious that a return to the spectroscopic methods would be desirable if it were possible, 1st, to increase the resolving power of our gratings; 2d, to concentrate all the light in one spectrum.

It is well known that the resolving power of a grating is measured by the product of the number of lines by the order of the spectrum. Attention has hitherto been confined almost exclusively to the first of these factors, and in the large six-inch grating of Professor Rowland there are about one hundred thousand lines. It is possible that the limit in this direction has already been reached; for it appears that gratings ruled on the same engine with but half as many lines have almost the same resolving power as the larger ones. This must be due to the errors in spacing of the lines; and if this error could be overcome, the resolving power could be augmented indefinitely.

In the hope of accomplishing something in this direction, together with Mr. S. W. Stratton, I constructed a ruling engine in which I make use of the principle of the interferometer in order to correct the screw by means of light-waves from a homogeneous source. This instrument (only a small model of a larger one now under construction) has already furnished rather good gratings of two inches ruled surface, and it seems not unreasonable to hope for a twelve-inch grating with almost theoretically accurate rulings.

As regards the second factor, the order of the spectrum observed, but little use is made of orders higher than the fourth, chiefly on account of the faintness of the light. It is true that occasionally a grating is ruled which gives exceptionally bright spectra of the second or third order, and such gratings are as valuable as they are rare; for it appears

that this quality of throwing an excess of light in a particular spectrum is due to the character of the ruling diamond, which cannot be determined except by the unsatisfactory process of trial and error.

If it were desired to produce rulings which should throw the greater part of the incident light in a given spectrum, we should try to give the rulings the form shown in section in Figure 1.

FIGURE 1.

I am aware of the difficulties to be encountered in the attempt to put this idea into practical shape, and it may well be that they are in fact insurmountable; but in any case it seems to be well worth the attempt.

Meanwhile the idea suggested itself of avoiding the difficulty in the following way.

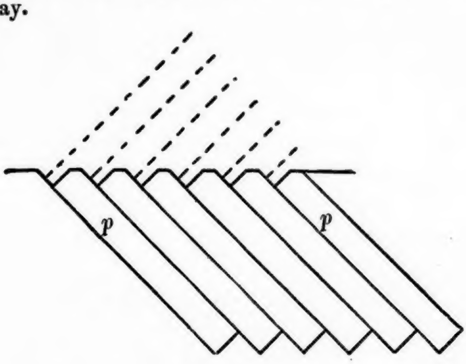

FIGURE 2.

Plates of glass (*pp*, Fig. 2) accurately plane-parallel and of the same thickness, are placed in contact, as shown in the figure. If the thicknesses were exactly the same, and were it not for variations in the thickness of the air-films between the plates, the retardations of the pencils reflected by the successive surfaces would be exactly the same, the reflected waves would be in the same conditions as in the case of a reflecting grating, — except that the common retardation is enormously greater.

The first condition is not very difficult to fulfil; but, in consequence of dust particles which invariably deposit on the glass surfaces, — in spite of the greatest possible precaution, — it is practically impossible to insure a perfect contact, or even constancy, in the distances between surfaces.*

If now instead of the retardation by reflection we make use of the retardation by transmission through the glass, the difficulty disappears almost completely. In particular the air-films are compensated by equivalent thicknesses of air outside, so that it is no longer necessary that their thickness should be constant. Besides, the accuracy of parallelism and of thickness of the glass plates necessary to insure good results is now only one fourth of that required of the reflection arrangement.

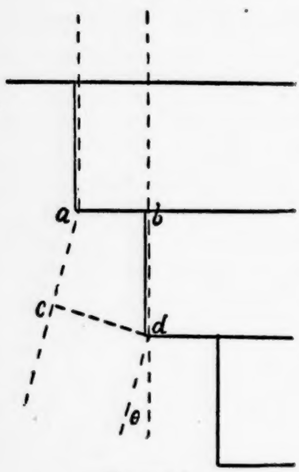

FIGURE 3.

In Figure 3 let $ab = s$, the breadth of each pencil of rays; $bd = t$, the thickness of each element of the echelon; θ, the angle of diffraction; α, the angle adb; m, the number of waves of length λ corresponding to the common difference of path of the successive elements. The difference of path is $m\lambda = \mu t - ad$. $ac = \frac{t}{\cos\alpha}\cos(\alpha + \theta)$; or, since θ is always very small,

$$ac = \frac{t}{\cos\alpha}(\cos\alpha - \theta\sin\alpha) = t(1 - \theta\tan\alpha),$$

and

$$m\lambda = (\mu - 1)t + s\theta. \qquad \text{I.}$$

To find the angle corresponding to a given value $d\lambda$, differentiate for λ, and we find $\frac{d\theta}{d\lambda} = \frac{1}{s}\left(m - t\frac{d\mu}{d\lambda}\right)$.

Putting in this expression the approximate value of $m = (\mu - 1)\frac{t}{\lambda}$, we have

* Nevertheless I have succeeded with ten such plates, silvered on their front surfaces, in obtaining spectra which, though somewhat confused, were still pure enough to show phenomena such as the Zeeman effect, the broadening of lines by pressure, etc.; but evidently the limit had been nearly reached.

$$\frac{d\theta}{d\lambda/\lambda} = \left[(\mu - 1) - \lambda\frac{d\mu}{d\lambda}\right]\frac{t}{s} = b\frac{t}{s}. \qquad \text{II.}$$

For the majority of optical glasses b varies between 0.5 and 1.0.

The expression II. measures the dispersion of the echelon. To obtain the resolving power, put $e = d\lambda/\lambda$ for the limit. For this limiting value the angle θ will be $\lambda/n\,s$, where n is the number of elements; hence $n\,s =$ the effective diameter of the observing telescope. Substituting these values, we find

$$e = \frac{\lambda}{b\,n\,t}. \qquad \text{III.}$$

To obtain the angular distance between the spectra, differentiate I. for m; we find

$$\frac{d\theta}{dm} = \frac{\lambda}{s}; \text{ or, putting } d\,m = \text{unity,}$$

$$d\theta_1 = \frac{\lambda}{s}. \qquad \text{IV.}$$

The quantity $d\lambda/\lambda = E$ corresponding to this is found by substituting this value of $d\theta$ in II., whence

$$E = \frac{\lambda}{b\,t}. \qquad \text{V.}$$

Hence the limit of resolution is the nth part of the distance between the spectra.

This fact is evidently a rather serious objection to this form of spectroscope. Thus, in observing the effect of increasing density on the breadth of the sodium lines, if the broadening be of the order of $\lambda/b\,t$ the two contiguous spectra (of the same line) will overlap. As a particular case, let us take $t = 7$ mm., $E = \frac{1}{17000}$. It will be impossible to examine lines whose breadth is greater than the fourteenth part of the distance between the D lines. It is evidently advantageous to make t as small as possible.

Now the resolving power, which may be defined by $\frac{1}{e}$ is proportional to the product $n\,t$. Consequently in order to increase it as much as possible it is necessary to use thick plates, or to increase their number. But in consequence of the losses by the successive reflections, experience shows that this number is limited to from 20 to 35 plates, any excess not contributing in any important degree to the efficiency.

I have constructed three echelons, the thickness of the plates being

7 mm., 18 mm., and 30 mm. respectively, each containing the maximum number of elements, — that is, 20 to 35, and whose theoretical resolving powers are therefore of the order of 210000, 540000, and 900000, respectively. In other words, they can resolve lines whose distances apart is the two-hundredth, the five-hundredth, and the nine-hundredth of the distance between the D lines.

Consequently the smallest of these echelons surpasses the resolving power of the best gratings, and what is even more important, it concentrates all the light in a single spectrum.

The law of the distribution of intensities in the successive spectra is readily deduced from the integral

$$A = \int_{s/2}^{s/2} \cos p x \, dx, \quad \text{where} \quad p = \frac{2\pi}{\lambda}\theta.$$

Hence

$$I = A^2 = \frac{\sin^2 \pi \frac{s}{\lambda}\theta}{\left(\pi \frac{s}{\lambda}\theta\right)^2}.$$

This expression vanishes for $\theta = t\lambda / s$, which is also the value of $d\theta_1$, the distance between the spectra.

Hence in general there are two spectra visible as indicated in Figure 4.

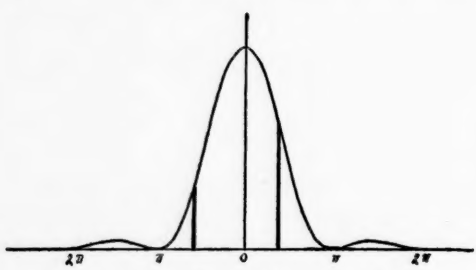

Figure 4.

By slightly inclining the echelon, one of the spectra is readily brought to the centre of the field, while the adjacent ones are at the minima, and disappear. The remaining spectra are practically invisible, except for very bright lines.

As has just been indicated, the proximity of the successive spectra of one and the same line is a serious objection, and as this proximity depends

on the thickness of the plates — which for mechanical reasons cannot well be reduced below 5 or 6 mm. — it is desirable to look to other means for obviating the difficulty, among which may be mentioned the use of a liquid instead of air.

In this case Formula II. becomes

$$\frac{d\theta}{d\lambda/\lambda} = \frac{t}{s}\left[\frac{1}{\mu_1}(\mu - \mu_1) - \lambda\frac{d(\mu - \mu_1)}{d\lambda}\right] = c\frac{t}{s}$$

and Formula IV. becomes

$$\frac{d\theta}{dm} = \frac{\lambda}{\mu_1 s}.$$

Repeating the same operations as in the former case, we find:

$$e = \frac{\lambda}{nct},$$

and

$$E = \frac{\lambda}{ct}.$$

The limit of resolution is still the n'th part of the distance between the spectra, but both are increased in the ratio b/c.

Suppose for instance the liquid is water. Neglecting dispersion the factor would be 3.55. Hence the distance between the spectra will be increased in this proportion, but the limit of resolution will also be multiplied by this factor. But as there is now a surface water-glass which reflects the light, the loss due to this reflection will be very much less, so that it will be possible to employ a greater number of elements, thus restoring the resolving power. At the same time the degree of accuracy necessary in working the plates is 3.55 times less than before.

For many radiations the absorption due to thicknesses of the order of 50 cm. of glass would be a very serious objection to the employment of the transmission echelon. I have attempted, therefore, to carry out the original idea of a reflecting echelon, and it may be of interest to indicate in a general way how it is hoped the problem may be solved.

Among the various processes which have suggested themselves the following appear the most promising.

In the first a number of plates (20 to 30), of equal thickness, are fastened together as in Figure 5, and the surfaces A and B are ground and polished plane and parallel. They are then separated and placed on an inclined plane surface, as indicated in Figure 6.

If there are differences in thickness of the air-films, the resulting differences in the height of the plates will be less in the ratio tan α.

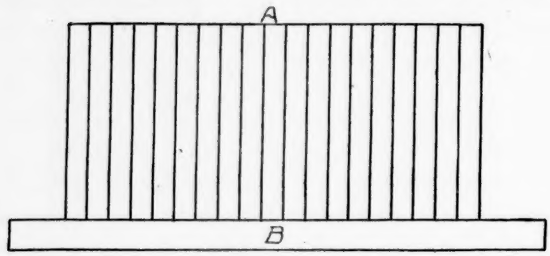

FIGURE 5.

An error of λ/n may be admitted for each plate, — even in the most unfavorable case in which the errors all add; and consequently the admissible errors in the thickness of the air-films may be of the order $\lambda/n\alpha$. For instance, for 20 plates the average error may be a whole wave-length if the inclination α is $\frac{1}{20}$. As there is always a more or

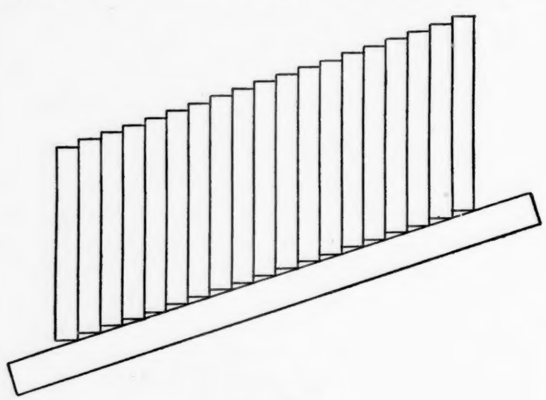

FIGURE 6.

less perfect compensation of the errors, the number of plates or the inclination may be correspondingly greater. Accordingly, it may be possible to make use of 50 elements, and the plane may be inclined at an angle of 20° to 30°. It would be necessary in this case, however, to use a rather large objective. Possibly this may be avoided by cutting the surface A to a spherical curvature, thus forming a sort of concave echelon.

FIGURE 7.

The second process differs from the first only in that each plate is cut independently to the necessary height to give the required retardation. The first approximation being made, the plates are placed on a plane surface, as in Figure 7.

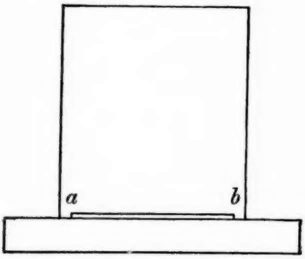

FIGURE 8.

The projections *a* and *b* (Fig. 8) are then ground and polished until the upper surfaces are all parallel, and the successive retardations equal. The parallelism as well as the height is verified by means of the interferometer.

These processes are, it is freely conceded, rather delicate, but preliminary experiments have shown that with patience they may be successful.